SYMPOSIUM OP'24

ADVANCING OCEAN PREDICTION SCIENCE FOR SOCIETAL BENEFITS

SASIP

Tailoring data assimilation to discontinuous Galerkin models

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Introduction

As part of the Scale-Aware Sea Ice Project (SASIP [1]) a new sea ice model, called *neXtSIM$_{DG}$*, is being developed. One of its novel features is that it uses a discontinuous Galerkin (DG) solver for the ice dynamics. It is envisaged that the model will be endowed with a data assimilation (DA) system. This system will use observations to bring the model state closer to the truth. In this work we will look whether it is possible to exploit the particular structure of a DG model to improve DA performance. In particular, we try to answer to following three questions:

- **Is it possible to assimilate multiple observations per grid cell in a Galerkin model, thus reducing the need for the formation of superobservations from dense (satellite) observation sets?**
- **Can DA also improve the estimate of the spatial derivative of the model fields in Galerkin models?**
- **Can the polynomial basis of the DG model be used to develop a scale-dependent ensemble localisation scheme?**

Discontinuous Galerkin

- A discontinuous Galerkin model approximates the solution in grid cell n as
$$u(x,t)|_n = \sum_k c_{kn}(t)\, p_k \circ \varphi_n(x)$$
with $u(x,t)|_n$ the possible multivalued model solution restricted to grid cell n, p_k the kth basis polynomial, φ_n a coordinate transform from cell n to the domain of the polynomials, c_{kn} the Galerkin coefficient for the cell and polynomial.
- Different polynomial families can be used for the p_k. E.g. Legendre or Lagrange polynomials.
- ODEs for c_{kn} can be found by inserting the expression for u in
$$\int_{D_n} p_k \circ \varphi_n(x) \left(\frac{\partial A(u(x,t),t)}{\partial t} - L(u(x,t),t) \right) dx = 0$$
for all n and k and solving the finite-dimensional system thus generated.

Experimental setup

A synthetic 1D twin experiment in a periodic domain of length *L=8000 km* is employed to answer the research questions. Following Liu & Rabier[2], an artificial truth, artificial observations and 40 ensemble members are generated from a given spectrum. These members are projected on DG spaces of different orders (yellow in Fig. 1). These projections are then interpolated to observation locations. As reference, the members are also evaluated on a 79-points (blue in Fig. 1). Linear interpolation is then used to interpolate from the points to the observation locations. The latter setup is referred to as the 'nodal' case. Observations are assimilated by ETKF[3] using the ensemble to estimate the forecast covariance.

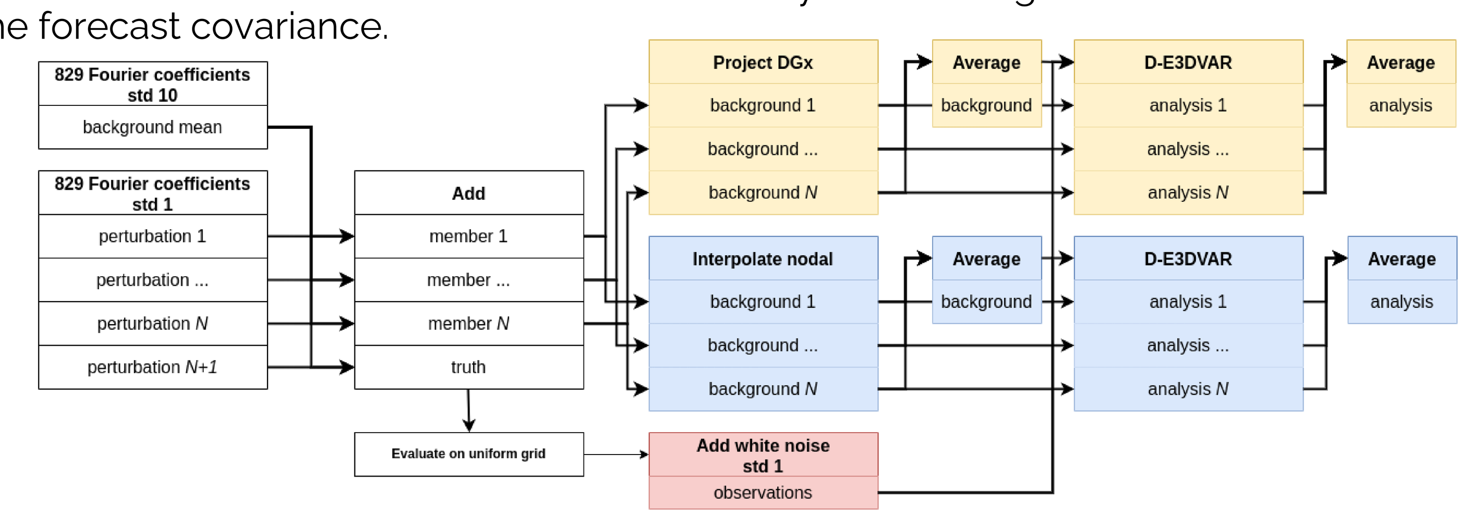


Figure 1: procedure to generate the artificial truth, artificial observations, ensembles and analysis.

Assimilated observation density

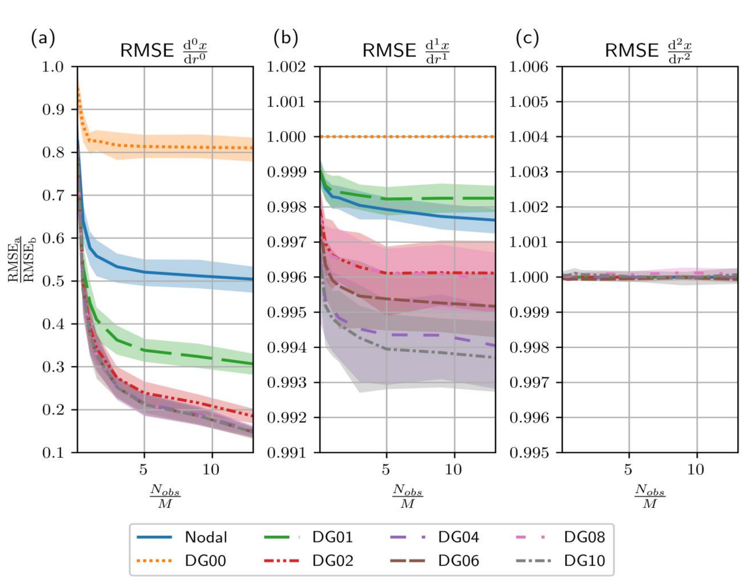


*Figure 2: RMS error ratio as function of the observation density for different DG orders with a **red k^{-4} error spectrum**.*

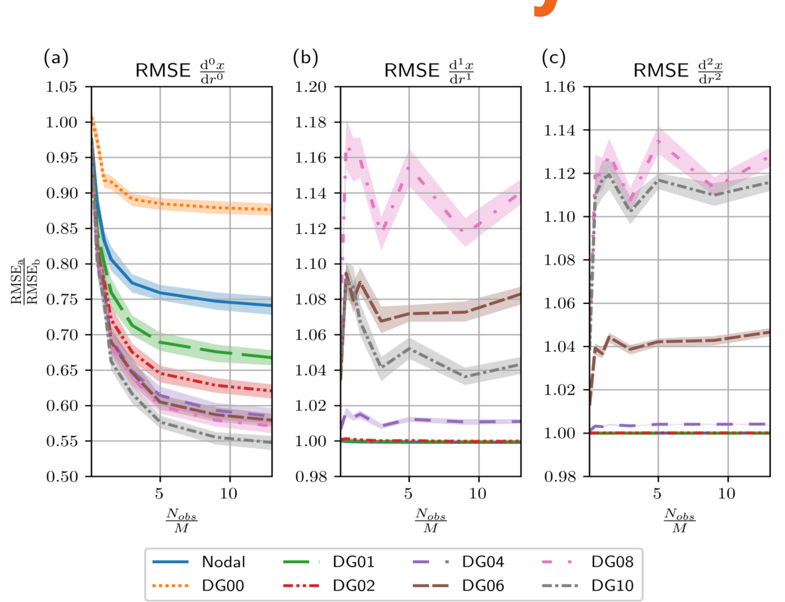


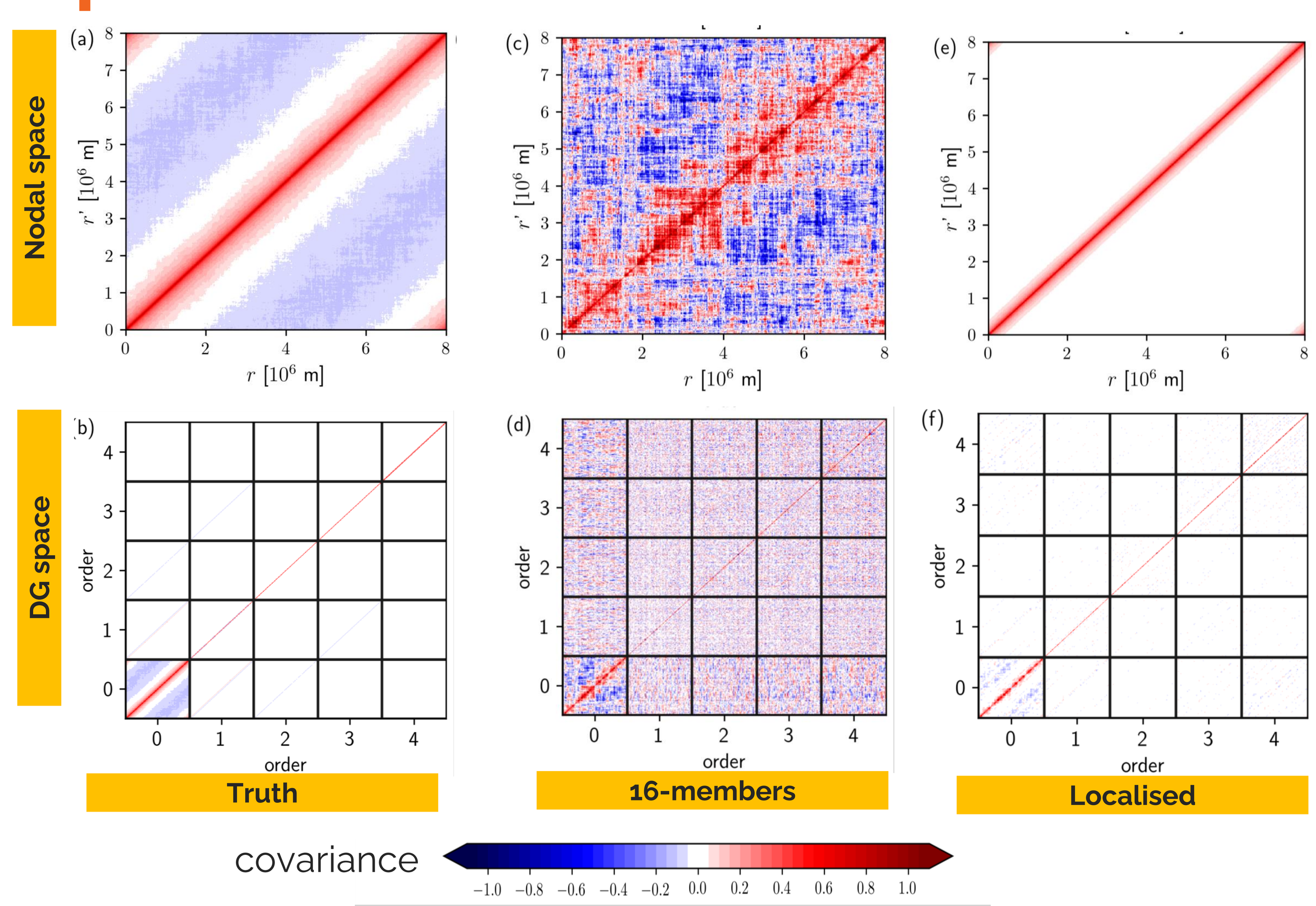
*Figure 3: as figure 2 but now for a **pink k^{-1} error spectrum**.*

The spatial root-mean square error (RMSE) between the ensemble mean and the artificial truth before/after DA is calculated for different number of observations per grid cell. Results in Fig. 2 and Fig. 3 show that:

- Assimilation of high-density observations is more effective if the error spectrum is redder (Fig. 2a vs. Fig. 3a).
- Assimilation of high-density observations benefits from the use of high-order DG schemes (Fig. 2a). Benefits of going to higher-order are less pronounced when the error spectrum is redder (Fig. 3a).
- Use of higher-order DG schemes in DA does not improve the derivatives of the model field (Fig. 2b,c and 3b,c).

Scale-dependent localisation

Figure 4: (a,b) true covariance; (c,d) estimate of the same covariance from 16 ensemble members; (e,f) same estimate after localisation. (top row) covariance between points on a 1D grid; (bottom row) covariance between DG coefficients.



When the covariance (Fig. 4a,b) is estimated from a small number of ensemble members (16) spurious correlations occur (Fig. 4c,d). To remove these spurious correlations, covariances are multiplied with a localisation factor (Fig. 5) calculated using the optimal localisation[4] approach.

- The covariances between higher-order DG coefficients are weak are near-diagonal (Fig. 4b).
- Length scales for the localisation factors are smaller for higher-order coefficients than for the 0th-order coefficient (Fig. 5). This because the DG polynomials act as a band-pass filter (Fig. 6).
- Scale independent localisation completely removes genuine negative correlations (Fig. 4e) whereas with scale dependent localisation part of it remains (Fig. 4f).
- Scale dependent approach results in smaller matrix error than scale independent localisation (not shown).

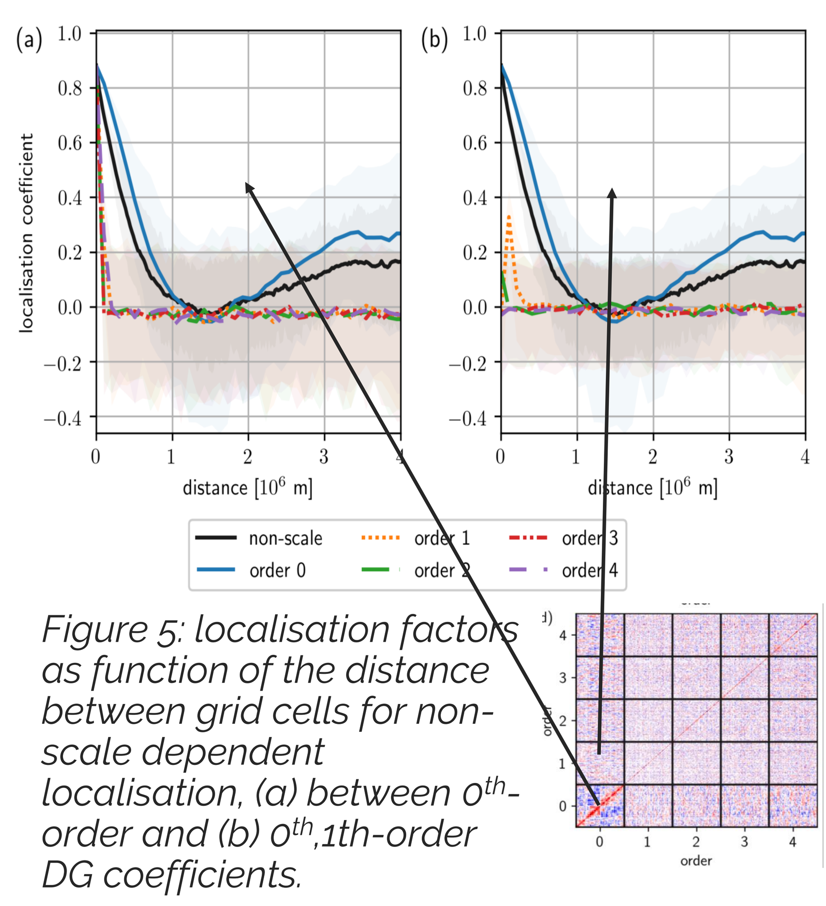


Figure 5: localisation factors as function of the distance between grid cells for non-scale dependent localisation, (a) between 0th-order and (b) 0th,1th-order DG coefficients.

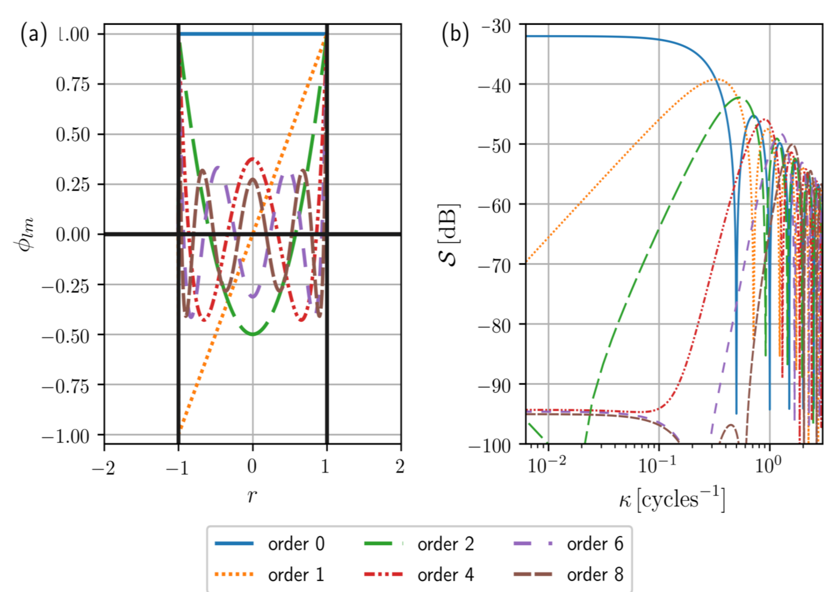


Figure 6: (a) Legendre polynomials of different orders in a grid cell. (b) Spectrum of the different Legendre polynomials as function of the wavenumber.

Conclusions

- **In a DG-model multiple (~5) observations can be assimilated effectively per grid. Especially, if the error spectrum is red.**
- **Assimilating multiple observations per grid cell does not improve the derivatives of the field.**
- **When a Legendre basis is used, the DG coefficients can be used to succesfully achieve scale-dependent localisation at little additional cost.**

References:
1. https://sasip-climate.github.io/
2. Liu, Z. Q., & Rabier, F. (2002). The interaction between model resolution, observation resolution and observation density in data assimilation: A one-dimensional study. *Quarterly Journal of the Royal Meteorological Society: A journal of the atmospheric sciences, applied meteorology and physical oceanography*, *128*(582), 1367-1386..
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