

Dynamical systems for remote validation of very high-resolution ocean models

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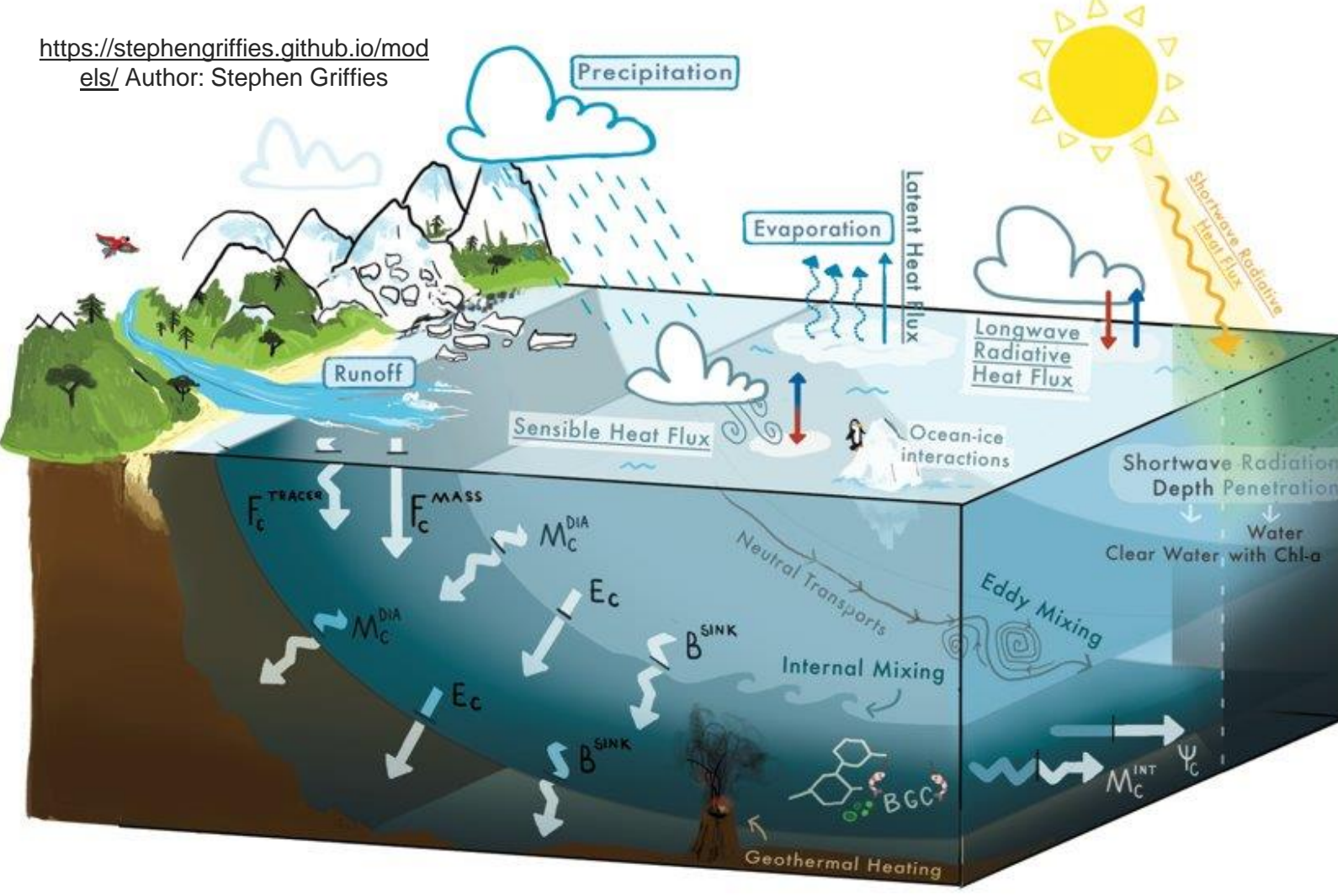
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1. Introduction



1. Ocean modeling presents significant challenges due to the multitude of complex physical phenomena that must be accurately represented. Furthermore, validating these models is particularly difficult in coastal areas, where in situ data is often limited by sparse observational networks and the constraints of specialized equipment deployment in select regions.

1. Our approach combines satellite observations with computational experiments using the Regional Ocean Modeling System (ROMS) [3],

1. We establish connections between satellite data and model outputs through dynamical systems analysis, enhancing our understanding of ocean dynamics in areas with limited direct observations.

3. Methodology

Base Solution

Parameter	Value
Horizontal Viscosity $A_{H,V}$	$2 m^2/s$
Horizontal Diffusivity $A_{T,S}$	$2 m^2/s$
Bottom Drag factor C_b^d	0.003 (non-dimensional)
Sponge factor	1 (non-dimensional)
Wind drag factor C_s^d	$6.0 \cdot 10^{-5}$ (non-dimensional)
Bathymetry	NAVIONICS
Boundary conditions	Clamped

Category	Experiment	Parameters
C1: Horizontal Mixing Coefficient	Exp 1	$A_{H,V} = A_{T,S} = 0.2 m^2/s$
	Exp 2	$A_{H,V} = A_{T,S} = 5 m^2/s$
C2: Bottom Drag Coefficient	Exp 1	$C_b^d = 0.03$ (non-dimensional)
	Exp 2	$C_b^d = 3 \cdot 10^{-5}$ (non-dimensional)
C3: Wind stress	Exp 1	$C_s^d = 6 \cdot 10^{-4}$
	Exp 2	$C_s^d = 6 \cdot 10^{-7}$
C4: Modified Bathymetry	Exp 1	GEBCO data (Fig. 7.1 b))
C5: Sponge at Boundary	Exp 1	$6 \cdot A_{H,V,T,S}$
C6: Mixed Radiation-Nudging BC	Exp 1	No sponge, $\tau_{nud} = 10$ days.
	Exp 2	Sponge, $\tau_{nud} = 10$ days

To establish connections between satellite data and model outputs, we employ a technique from the field of dynamical systems known as the **Lagrangian Descriptor** or **M function**. This method enables the extraction of **Lagrangian Coherent Structures (LCS)**, which are time-dependent material surfaces that separate fluid paths with distinct behaviors [1].

$$M(x_0, t_0, \tau) = \int_{t_0-\tau}^{t_0+\tau} \|v(x(t), t)\| dt$$

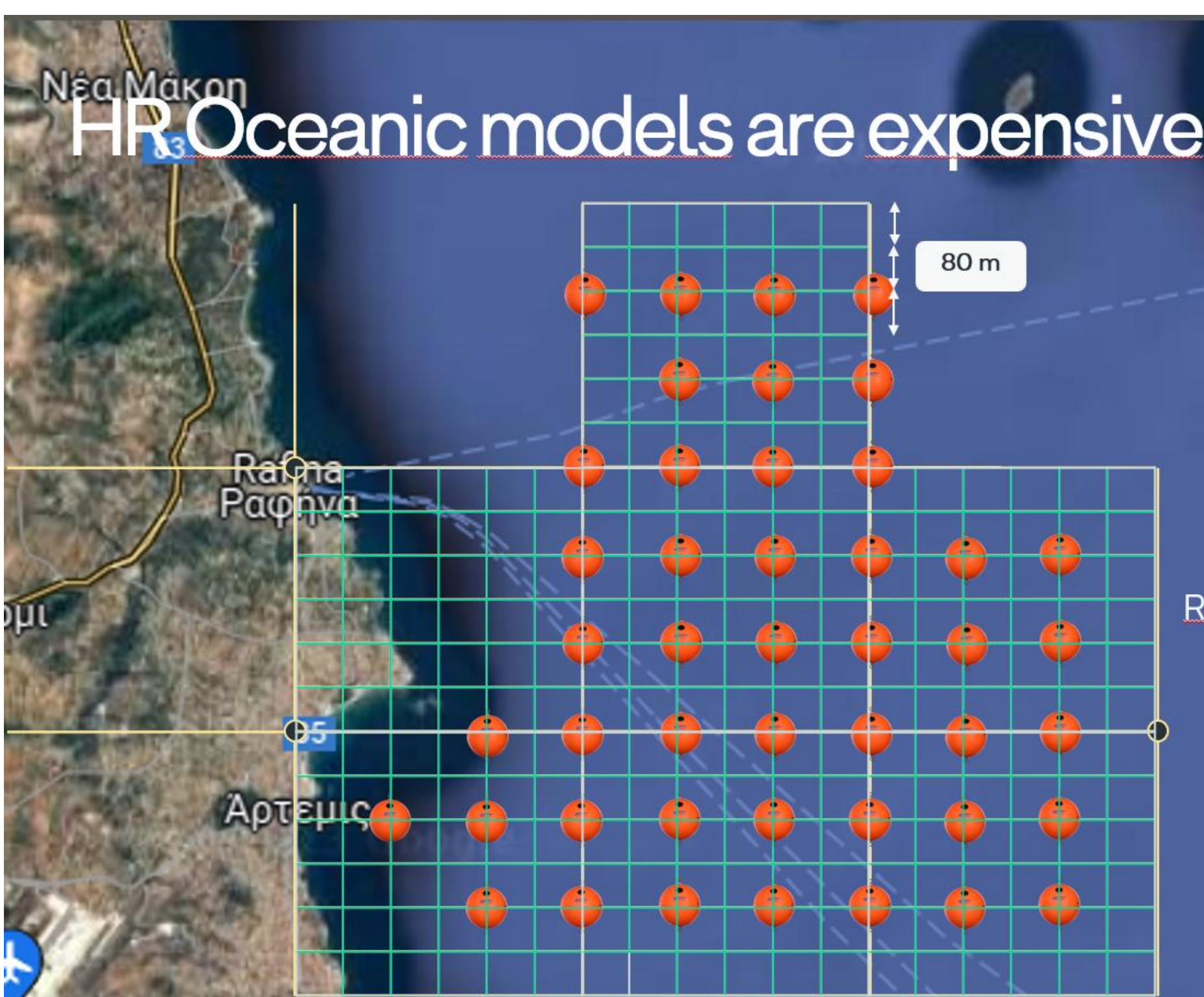
$$= \int_{t_0}^{t_0+\tau} \|v(x(t), t)\| dt + \int_{t_0-\tau}^{t_0} \|v(x(t), t)\| dt$$

A significant mathematical property of these Lagrangian patterns is their **robustness** against velocity field perturbations, in contrast to the comparison of individual trajectories. This characteristic allows for a more effective characterization of model-produced velocity fields by:

1. Differentiating outputs with distinct Lagrangian signatures
2. Identifying similarities between outputs that present comparable Lagrangian patterns

M measures the **arclength** of the curve traced by a trajectory, integrated from an initial condition **forwards** and **backwards** in time, when projected onto the phase space [2]

2. Study Area & Data



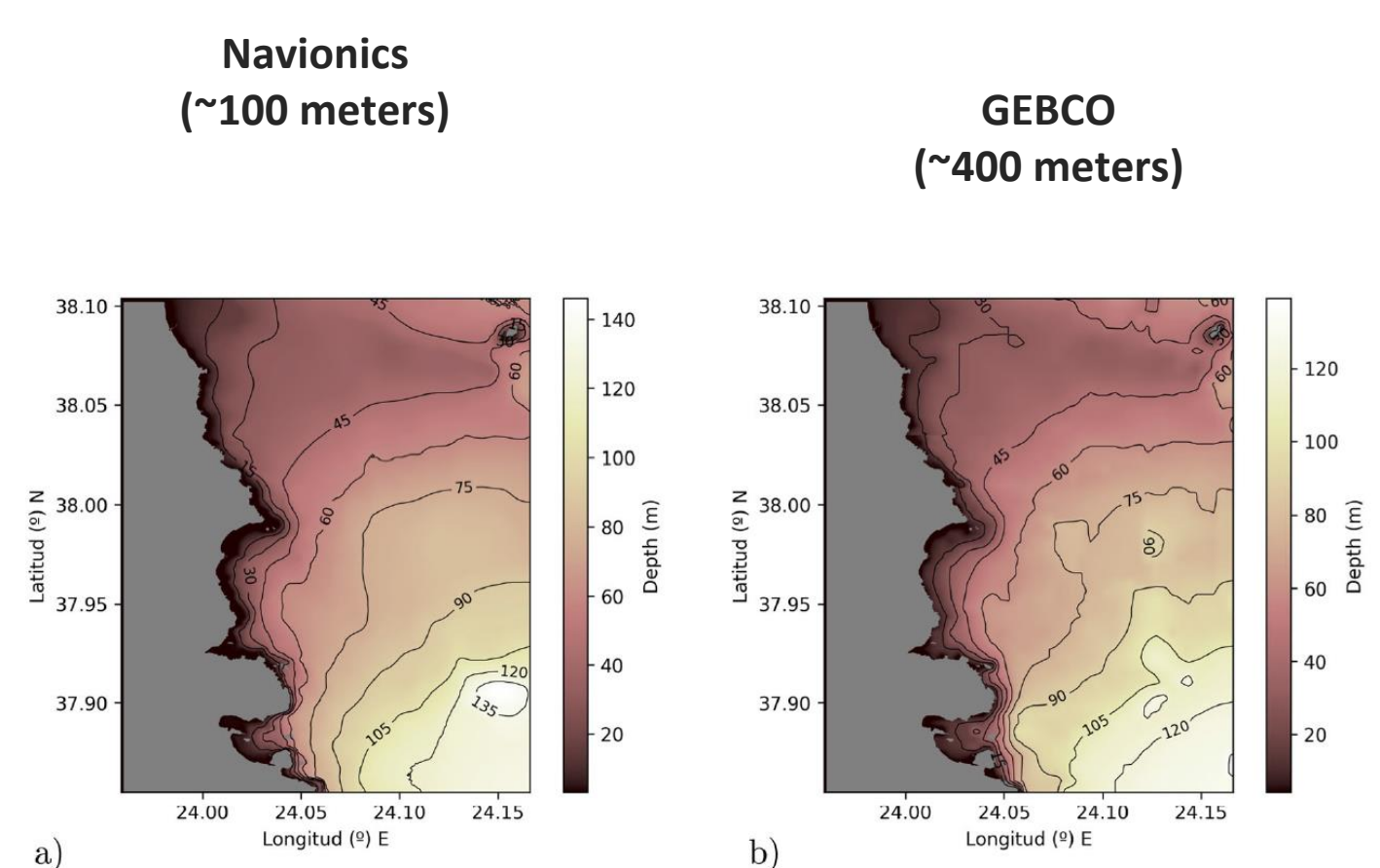
1. CMEMS data from Mediterranean Forecasting System (Med-Physics) Product

Variables: Sea Surface height, Temperature 3D, Salinity 3D, u and v 3D

2. ERAS data.

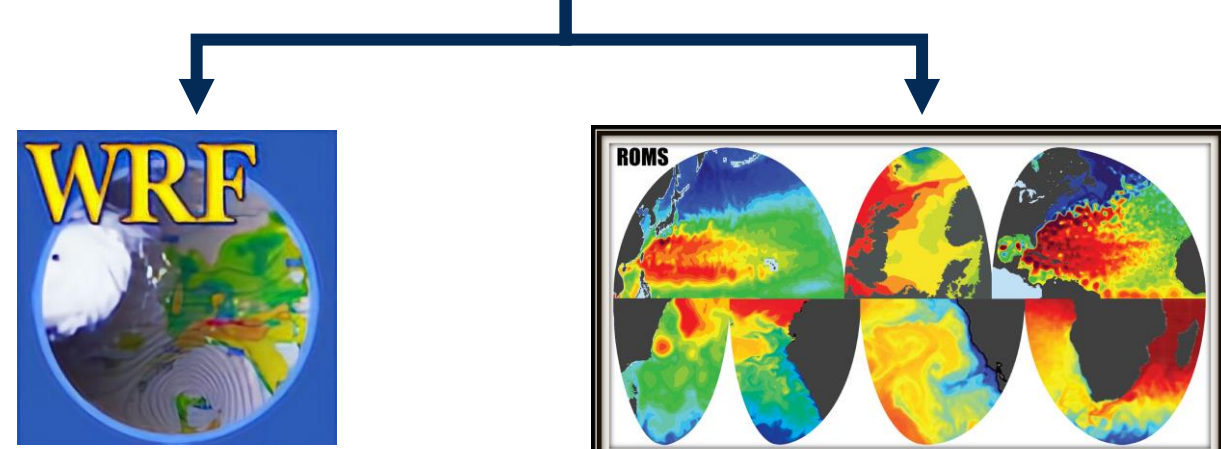
Variables: v10 and v10

3. Bathymetry data. Navionics maps and GEBCO



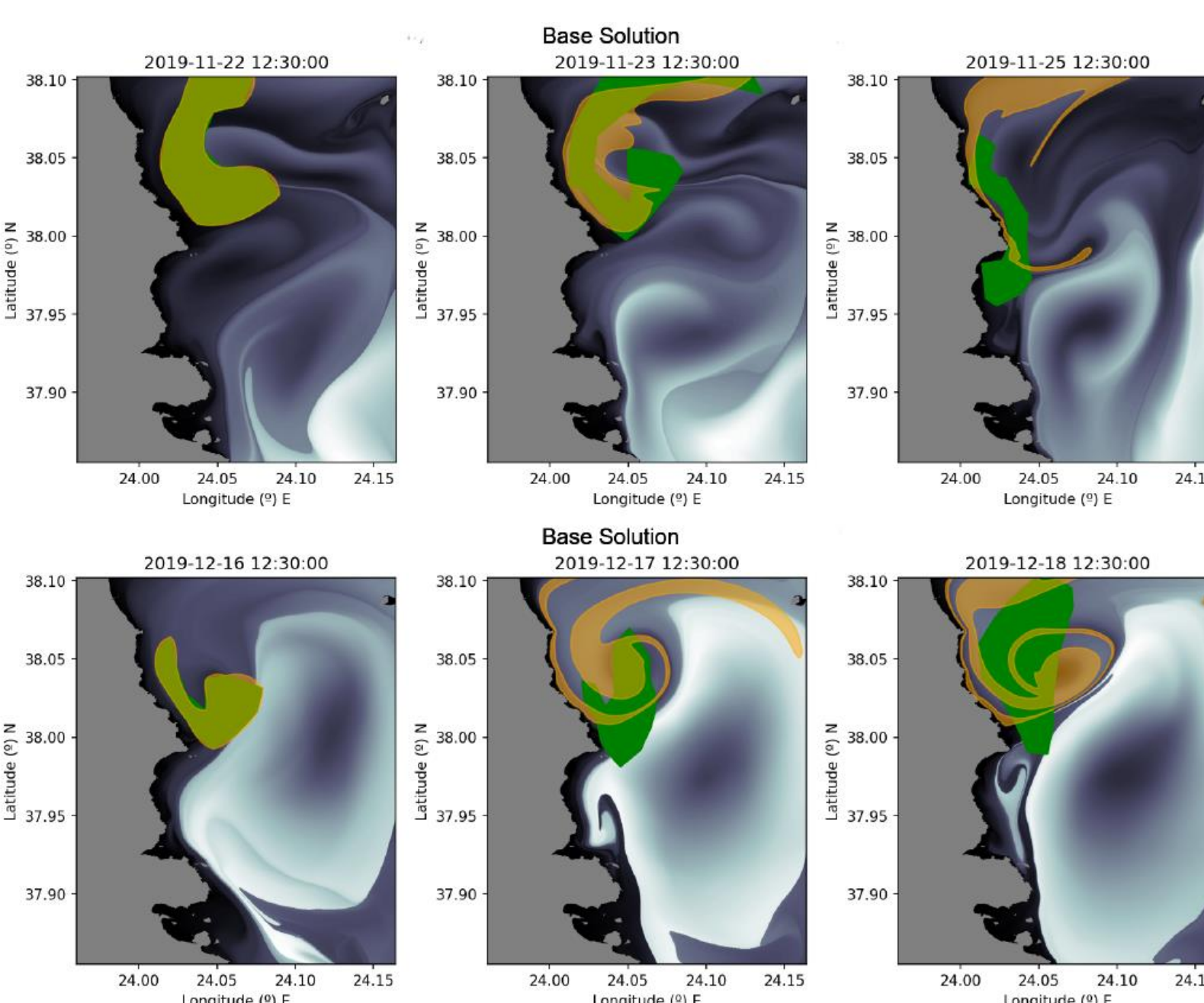
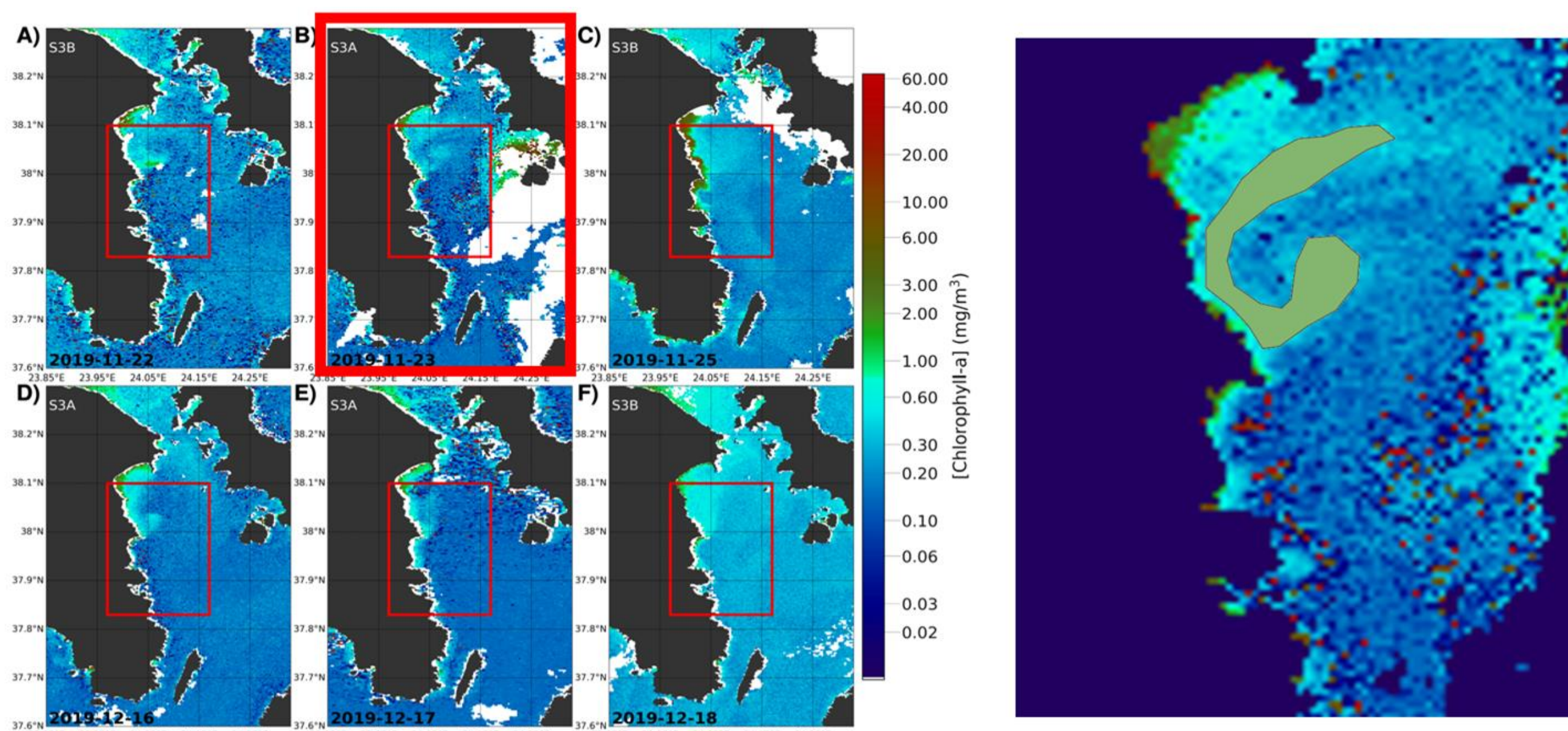
3. Methodology

Downscaling



Setup

Parameter	Value	Description
L_m	344	number of points in longitude direction
M_m	228	number of points in latitude direction
N	10	number of vertical (sigma) levels
h_{max}	146	maximum depth of the domain (metres)
h_{min}	0.29	minimum depth of the domain (metres)
θ_s	5.0	sigma coordinate surface stretching factor
θ_b	0.4	sigma coordinate bottom stretching factor
Δt	10	baroclinic time-step (seconds)
Δt_b	7	barotropic time-step (seconds)



1. Run for two periods in 2019 with available satellite images. System initialized before periods of interest to ensure convergence to pullback attractor

1. Extracted polygons from satellite images representing homogeneous chlorophyll distribution. Assumed these structures are purely advected by currents, serving as ground truth

1. Initialized chlorophyll blobs on first day of each period. Evolved blobs using velocity fields from base solution. Compared model-evolved blobs with satellite-observed polygons to assess model performance.

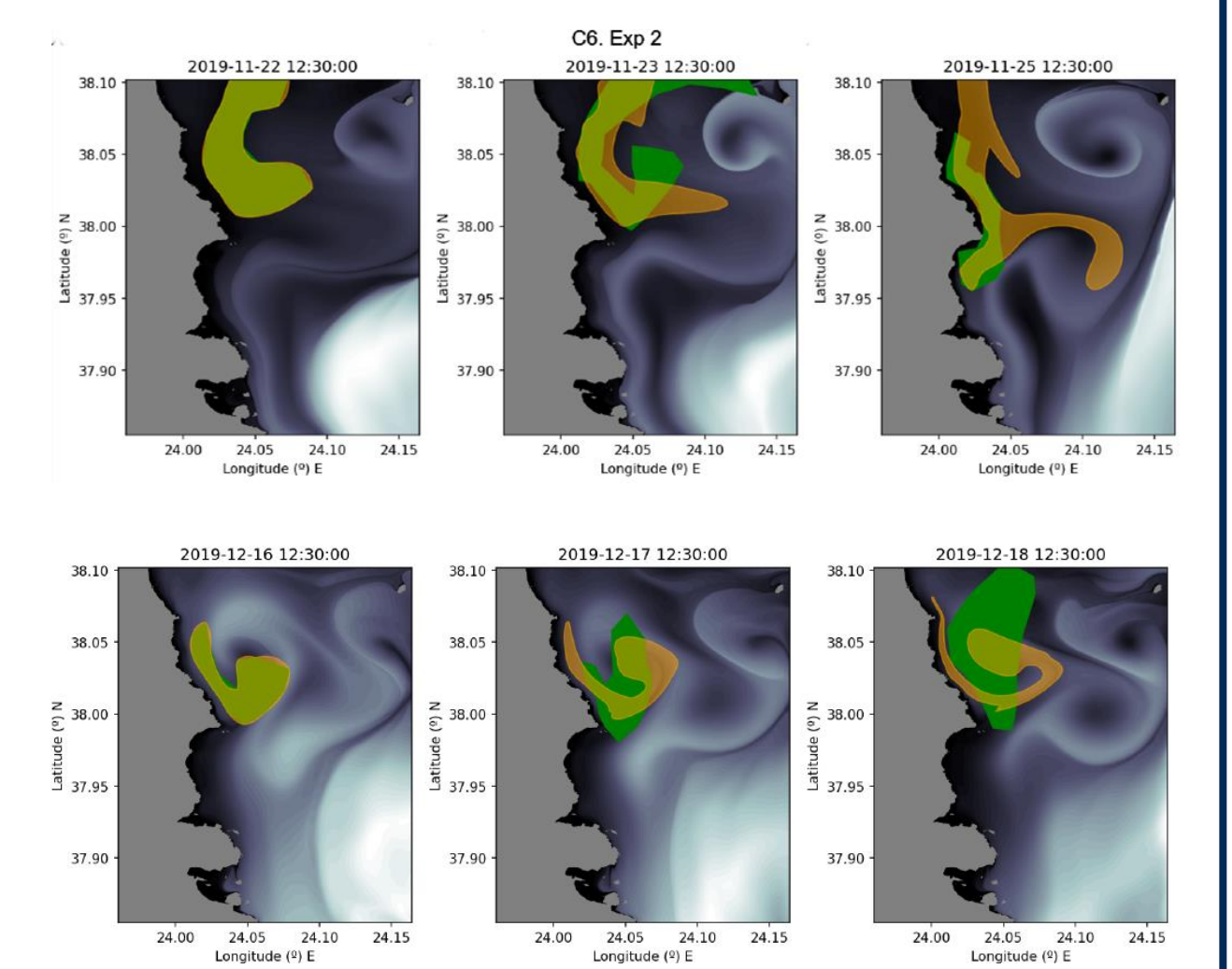
1. Compared model-evolved blobs with satellite-observed polygons. Calculated backward M for $\tau = 3$ days to highlight attracting material curves of the flow. Assessed consistency between these features and brown blob evolution. Measured overlap between green and brown blobs as performance indicator.

$$A = \frac{\text{Area}_{\text{intersection}}}{\text{Area}_{\text{truth}}}$$

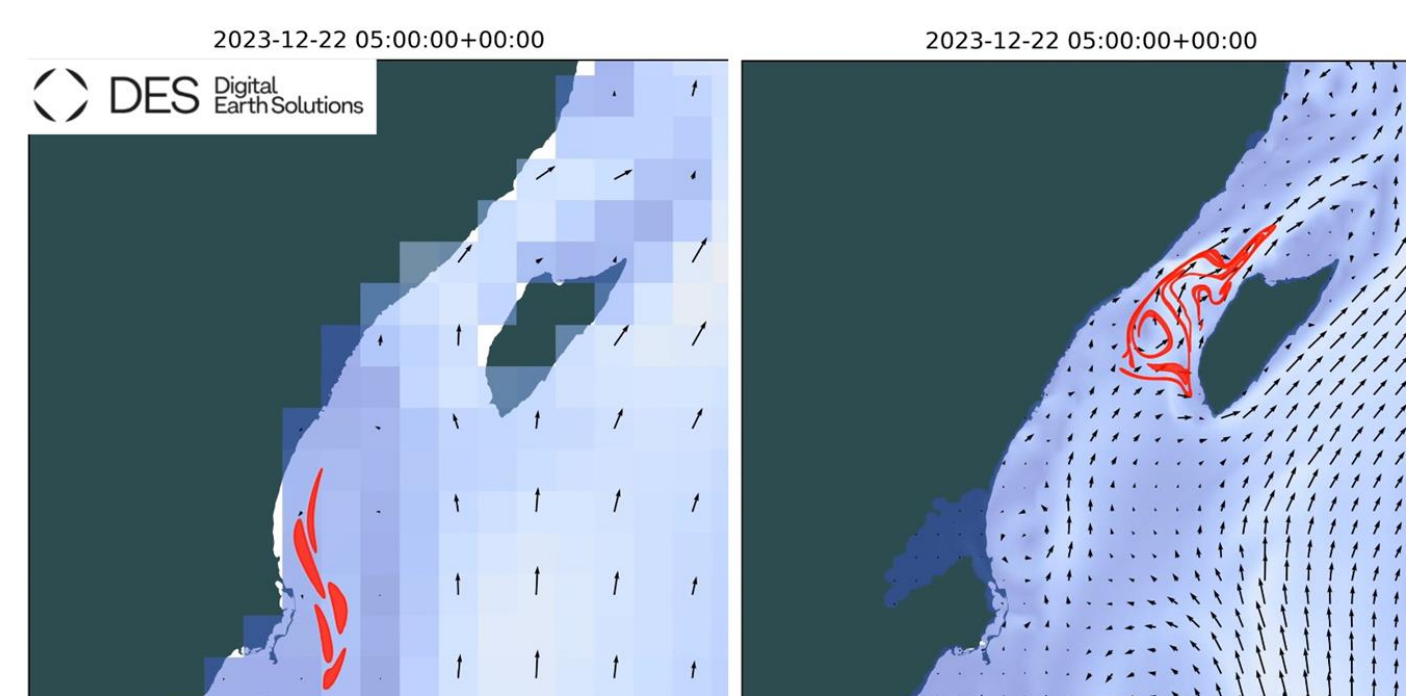
- A close to 1: good performance
- A close to 0: poor performance

Experiments Results

Experiments	Avg First Period	Avg Second Period	Total Avg
B.S.	0.2922	0.3195	0.3058
C1. Horizontal mixing exp 1	0.2569	0.2433	0.2501
C1. Horizontal mixing exp 2	0.3921	0.4345	0.4133
C2. Bottom drag exp 1	0.2660	0.4226	0.3443
C2. Bottom drag exp 2	0.3770	0.2980	0.3375
C3. Wind stress exp 1	0.4402	0.3194	0.3798
C3. Wind stress exp 2	0.2876	0.3321	0.3099
C4. Modified bathymetry exp 1	0.2273	0.1834	0.2054
C5. Sponge exp 1	0.2028	0.2762	0.2395
C6. Mixed radiation-nudging exp 1	0.4498	0.1998	0.3249
C6. Mixed radiation-nudging exp 2	0.6515	0.3871	0.5194



4. Conclusions



Our team successfully replicated and adapted a methodology to study sargassum flow in the Yucatan Peninsula, Mexico. This project, financed by ESA and Madrid I+D, aimed to address the pressing sargassum problem in the region.

References

- [1] García-Sánchez, G., Mancho, A.M., Wiggins, S.: A bridge between invariant dynamical structures and uncertainty quantification. Commun. Nonlinear Sci. Numer. Simul. **104**, 106016 (2022).
- [2] Mancho, A.M., Wiggins, S., Curbelo, J., Mendoza, C.: Lagrangian descriptors: a method for revealing phase space structures of general time dependent dynamical systems. Commun. Nonlinear Sci. Numer. Simul. **18**(12), 3530–3557 (2013).
- [3] Shchepetkin, A. F., McWilliams, J. C.: The regional oceanic modeling system (ROMS): A split-explicit, free-surface, topography-following-coordinate oceanic model. Ocean Modell. **9** (2005)